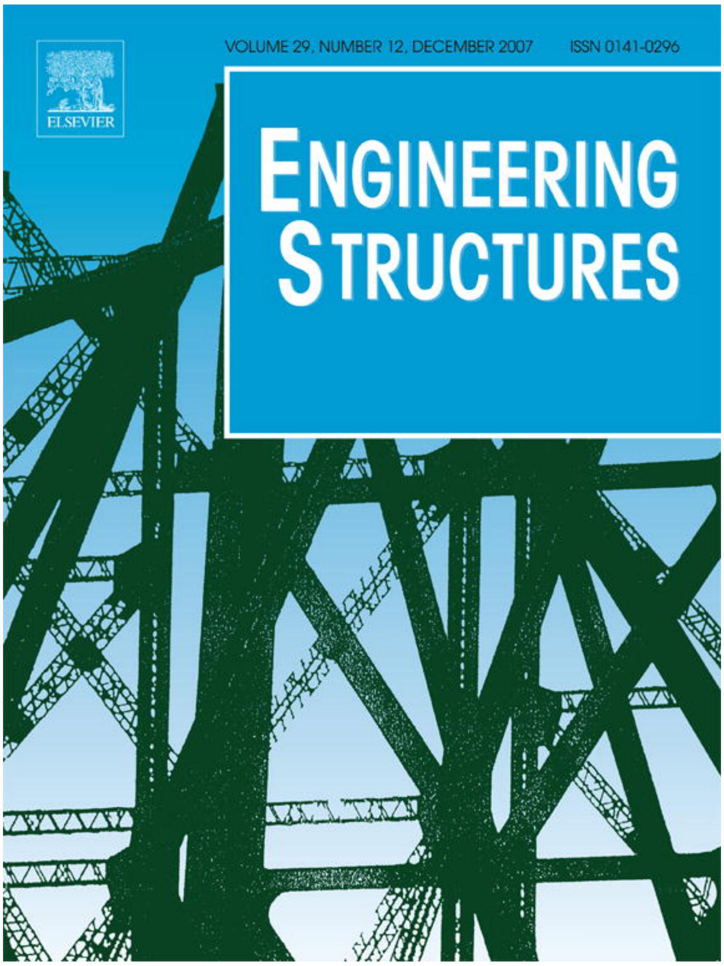
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Automated decision procedure for earthquake early warning

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Abstract

Early warning systems represent an effective tool for disseminating timely information about potentially catastrophic hazards to the public, emergency managers, and operators and users of transportation systems, industrial facilities, public buildings including schools and government offices, and private residences. This information gives the possibility of taking action to initiate mitigation or security measures before a catastrophic event occurs. Earthquake early warning technologies represent a promising application, especially in areas where faults are located in the proximity of an urbanized area, such as in Los Angeles and San Francisco, California. For earthquake early warning systems, automated procedures for taking action are necessary due to the short lead time available, including an automated decision procedure to decide when to activate mitigation or security measures based on the expected consequences of taking, or not taking, such actions. In this work, a decision procedure is presented that accounts for the uncertainty in the predicted ground shaking intensity at a site of a facility by deciding whether or not to take a mitigation action based on the probabilities of false and missed alarms. It uses a cost-benefit analysis of the consequences of making a wrong decision. It is shown how the method may be extended to other predictors, such as expected losses, that may be more effective parameters for rational decision making from the perspective of facility stakeholders.

Keywords: Earthquake early warning; Decision models; Cost-benefit analysis; Real-time loss estimation

1. Introduction

Early warning technologies can be a key component to mitigate the effects of potentially catastrophic natural hazards. A recent example of the need for timely warnings is given by the Asian tsunami disaster which occurred on 26 December 2004. Although early warning systems have long been developed for mitigation of natural hazards such as tsunamis, hurricanes and tornados, it is only recently that they have become feasible for seismic risk mitigation, which has prompted an increasing amount of research interest [1–12]. The main goal of an earthquake early warning system (EEWS) is to take some action to protect or reduce loss of life or to mitigate damage and economic loss. Examples of such measures are evacuation of buildings, shut-down of critical systems (nuclear reactors, industrial chemical processes, etc.) and stopping of high-speed trains. The basic hardware of an EEWS is illustrated in Fig. 1.

Effective early warning technologies for earthquakes are much more challenging to develop than for other natural hazards because warning times range from only a few seconds in the area close to a rupturing fault to a minute or so [1,3,4]; longer times will usually mean that the earthquake source is sufficiently distant from a site that the seismic waves will not pose a hazard at the site (a notable exception being the hazard in Mexico City from earthquake sources on the Pacific Coast subduction zone [5]). The type of mitigation measures that can be effectively activated depends on the amount of warning time available. Furthermore, the short warning times mean that to be effective, an EEWS must depend on automated procedures, including those for decision making about whether or not to activate mitigation measures; the time is too short to require human intervention when the event is first detected. As a result of the automation, careful attention must be paid to the design of a local EEWS at protected facilities.

Timeliness is often in conflict with the desire to have reliable predictions, which become more accurate as more seismic

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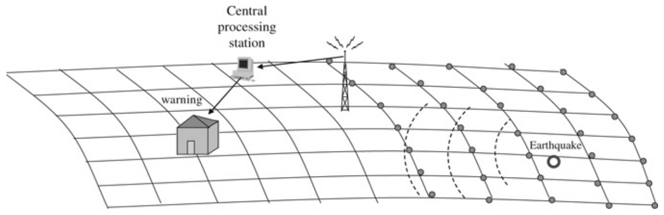

Fig. 1. The EEWS is composed of a network of seismic stations, a dedicated real-time data communication system, central processing system, broadcasting system and information receivers at the user's end. In some cases, this network may be a dedicated local one placed at some distance around a structure to provide data about incoming seismic waves. When a seismic event occurs, the stations close to the epicentral area are triggered by P-waves, then the ground motion data is recorded and sent by the communication system to the central processor, where, based on predictive models, an evaluation is made in real-time to predict the earthquake source parameters of the event. This information is provided to the user by a distribution network, and based on this early information, the user predicts a ground motion or structural performance parameter of interest for a facility. This parameter represents the predictor on which the decision whether to take a mitigation action or not is based.

sensor data is collected from the EEWS network. There is therefore an inevitable trade-off between the amount of warning time available and the reliability of the predictions provided by an EEWS. To investigate this trade-off, the expected consequences of the two alternatives of taking, or not taking, mitigation actions must be analyzed, taking into account the significant uncertainty in the predictions; in particular, a means of controlling the trade-off between false alarms and missed alarms is desirable. An automated real-time procedure for solving this trade-off is presented in this paper that determines the optimal time to activate the mitigation measure based on the increasing accuracy of the information supplied by the EEWS and the expected consequences of the alternative actions.

1.1. Basic principles of an EEWS

The fundamental principle on which an EEWS is based is that seismic waves travel through the Earth with a velocity that is much less than the velocity of the electromagnetic signals that can be transmitted by telephone line or radio to provide seismic information about the incoming event. In addition, seismic body waves can be identified as compression waves (primary waves or P-waves) and shear waves (secondary waves or S-waves) where the P-waves are characterized by a propagation velocity that is almost twice that of S-waves, which are stronger and which give almost horizontal ground motion at the base of a structure, in contrast to P-waves, so S-waves tend to be more damaging. The objective of an EEWS is to take advantage of the time interval between the detection of P-waves in the epicentral area and the arrival of S-waves in the area where the structure or facility is located; during this time seismic parameters are estimated from a sufficient length of the recorded waveform and then a warning is sent before the strong shaking initiates at the facility, as illustrated in Fig. 1.

The feasible warning time at a site of interest is given by:

$$T_w = T_s - T_p - T_r \tag{1}$$

$$T_r = T_d + T_{pr} \tag{2}$$

where T_s is the S-wave travel time from the point of initiation of the fault rupture to the site of interest; T_p is the P-wave travel time from the point of initiation of the fault rupture to the first station of the EEWS network recording the seismic event; T_r is the reporting time which comprises the time T_d needed by the detection system to trigger and record a sufficient length of waveforms to be able to assess the parameters of the event and the time T_{pr} to process the data. For the warning time T_w to be useful, it has to be greater than the time necessary for activation of the mitigation measure.

An initial alert signal may be sent to give the maximum amount of warning time when a minimum level of prediction accuracy has been reached. However, the prediction accuracy for the location and size of the event will continue to improve as more data is collected by the EEWS network. As time passes and the improved estimates and their uncertainty are received at the site of a facility equipped to take some mitigation action, a decision must be made as to whether or not to activate the mitigation measure. As mentioned before, this involves a trade-off between timeliness and reliability. Because of the uncertainties in the predicted parameters for the seismic event, it is possible that a wrong decision may be made. A decision model is therefore required to take into account the expected consequences of making a wrong decision. One must therefore consider the consequences of making false alarms and missed alarms.

Suppose that at the site of a facility, a mitigation measure is to be activated if a critical ground shaking intensity threshold is *predicted* to be exceeded at the site, based on the information received from an EEWS. The choice of critical threshold will depend, of course, on the vulnerability of the system to be protected. Assuming that the warning time is sufficient for activation of the mitigation measure, then based on the predicted parameters supplied by the EEWS from the first few seconds of P-wave observation, a decision has to be made of whether or not to activate the mitigation measure. In making this decision, two kinds of wrong decisions may be committed [13]:

- *Missed Alarm* (*or False Negative*): the mitigation action is not taken when it should have been;
- *False Alarm* (*or False Positive*): the mitigation action is taken when it should not have been.

The probability of each of these wrong decisions is denoted by:

P_{ma} = probability of missed alarm, that is, the probability of having critical threshold exceedance by the ground shaking but no activation of the mitigation measure is made;

P_{fa} = probability of false alarm, that is, the probability of having no threshold exceedance by the ground shaking but activation of the mitigation measure is made.

The tolerance of a missed alarm or false alarm is related to a trade-off between the benefits of a correct decision and the costs of a wrong decision and it could vary substantially, depending on the relative consequences of possible missed and false alarms. For example, the automated opening of a fire station door has minimal impact if the door is opened for a false alarm and so when designing the local EEWS, one could focus on reducing the probability of a missed alarm. On the contrary, an automated shut-down of a power plant because of a false alarm could cause problems over an entire city and involve expensive procedures to restore it back to its full-operational status. In this case, the EEWS must be designed to keep the probability of false alarms very low. In general, the automated decision process has to be designed with attention focused on the probability of false and missed alarms and a balance chosen based on a cost-benefit analysis. It could turn out that some mitigation measures are unacceptable to operate as a result of a high false or missed alarm rate. The decision procedure proposed in this paper, which is based on real-time estimates of the probability of wrong decisions (or alternatively based on a time-varying warning threshold exceedance, see Section 3.2), provides a mechanism to control the incidence of false and missed alarms. Since it is not possible to simultaneously reduce both of these, the decision making procedure can be used to control the trade-off between them.

The probability of a wrong decision is due to having only partial knowledge of the phenomenon, which leads to any prediction being accompanied by uncertainty. A key element of an EEWS is a better understanding of the parameters that play a fundamental role in this uncertainty, and hence a better understanding of the quality of the predictions on which the decision making is to be based. In the next section, an analysis is made of how the uncertainty in the predicted magnitude and location of the event affect the prediction of the ground shaking intensity, denoted by *IM* (intensity measure).

2. Uncertainty analysis of an EEWS operation

As illustrated in Fig. 1, an EEWS is composed of a network of seismic stations, a dedicated real-time data communication system, central processing system, broadcasting system and a local processing system at the user's end. In some cases, this network may be a dedicated local one placed at some distance around a special facility or structure to provide data about incoming seismic waves. When a seismic event occurs, the stations close to the epicentral area are triggered by P-waves, and then the ground motion data is recorded and sent by the communication system to the central processor, where, based on predictive models, an evaluation is made in real-time to predict the earthquake source parameters of the event.

This information is then provided to the user by a distribution network, where a dedicated processor at the site of the facility evaluates a performance parameter of interest. This parameter represents the predictor on which the decision whether or not to take a mitigation action is based. For the purpose of presenting the automated design procedure, the predictor is taken to be a ground-motion parameter *IM* that represents a measure of the shaking intensity at the site of the facility. It is to be predicted based on the earthquake source parameters sent by the central processing system of the EEWS. The parameter of interest for a facility could also be taken as some critical engineering demand parameter, such as the peak inter-story drift in a building or peak floor acceleration at the

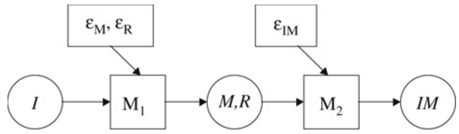

Fig. 2. The multi-component model for EEWS uncertainty propagation.

location of vulnerable equipment, or even economic loss, which is addressed in a later section.

The prediction model for the ground-motion parameters can be viewed as composed of two submodels, M_1 and M_2 (see Fig. 2). The earthquake prediction model, M_1, is part of the EEWS central processing system and it estimates the source parameters of the earthquake: its magnitude, M, and its epicenter location which, for a given site, implies its epicentral distance, R. These seismic parameters are estimated based on parameters, I, extracted from real-time measurements of the first few seconds of P-waves from the seismic event; for example, I is the predominant period in [3] and it is the observed ground motion ratio for the Virtual Seismologist method [14]. The ground-motion attenuation model, M_2, is likely to be part of the local processing system at a site and it predicts a ground-motion parameter (intensity measure *IM*), based on the magnitude and epicentral distance predicted by M_1. Based on the estimated value of *IM*, which is taken here to be the final outcome of the EEWS prediction process, along with the uncertainty of this estimate, the decision of whether or not to take some mitigation action is made.

The predictor is affected by uncertainty that is due to the errors ε_M, ε_R and ε_{IM} in the submodels M_1 and M_2 that propagate through to the output (see Fig. 2). The quality of the predictions of *IM* is fundamental for optimization of the decision making process and so it is important to estimate the uncertainty for the total prediction error in order to quantify the performance of the EEWS prediction process. For this purpose, consider the total prediction error, which is given by the difference between the actual and predicted intensity measures, IM and $\hat{IM}$:

$$\varepsilon_{\text{tot}} = IM - \hat{IM} \tag{3}$$

where $\hat{IM}$ is a function of the predicted magnitude and epicentral distance, $\hat{M}$ and $\hat{R}$, and IM is a function of the actual magnitude and epicentral distance, M and R, and ε_{IM}, the uncertain prediction error in the ground-motion attenuation model:

$$\begin{aligned} \hat{IM} &= f(\hat{M}, \hat{R}) \\ IM &= f(M, R) + \varepsilon_{IM} \end{aligned} \tag{4}$$

where function f represents the ground-motion attenuation model; for example, it could be of the following common form:

$$f(M, R) = c_1 + c_2 M + c_3 \log_{10} R \tag{5}$$

based on the assumption that IM is taken as the base-10 logarithm of a ground-motion parameter such as the peak ground acceleration (PGA) or acceleration response spectral ordinate (S_a) [15]. Note that $M = \hat{M} + \varepsilon_M$ and $R = \hat{R} + \varepsilon_R$, so the total prediction error may be expressed as:

$$\begin{aligned}\varepsilon_{\text{tot}} &= IM - I\hat{M} = f(\hat{M} + \varepsilon_M, \hat{R} + \varepsilon_R) + \varepsilon_{IM} - f(\hat{M}, \hat{R}) \\ &= g(\hat{M}, \hat{R}, \varepsilon_M, \varepsilon_R) + \varepsilon_{IM}. \end{aligned} \quad (6)$$

The functional form for g is defined explicitly here; for the attenuation model in Eq. (5), it is:

$$\begin{aligned}\varepsilon_{\text{tot}} &= c_2\varepsilon_M + c_3\varepsilon_{\log R} + \varepsilon_{IM} \\ &\approx c_2\varepsilon_M + \frac{c_3}{\hat{R}}\varepsilon_R + \varepsilon_{IM}\end{aligned} \quad (7)$$

where $\varepsilon_{\log R} = \log_{10} R - \log_{10} \hat{R} = \log_{10}(1 + \varepsilon_R/\hat{R}) \approx \varepsilon_R/\hat{R}$.

An appropriate probability model for ε_M, $\varepsilon_{\log R}$ and ε_{IM} is given by a Gaussian distribution [14,16]. For the attenuation model in Eq. (5), the total prediction error ε_{tot} then follows a Gaussian distribution with a variance that depends on the variances of the contributing prediction errors according to Eq. (7):

$$\begin{aligned}\sigma_{\text{tot}} &= \sqrt{c_2^2\sigma_M^2 + c_3^2\sigma_{\log R}^2 + \sigma_{IM}^2} \\ &\approx \sqrt{c_2^2\sigma_M^2 + \frac{c_3^2}{\hat{R}^2}\sigma_R^2 + \sigma_{IM}^2}\end{aligned} \quad (8)$$

under the assumption of probabilistic independence of the contributing errors, and with a mean given by:

$$\mu_{\text{tot}} = c_2\mu_M + c_3\mu_{\log R} + \mu_{IM}. \quad (9)$$

It is usually appropriate to take the mean to be zero.

If the attenuation model is more complicated than in Eq. (5), Monte Carlo simulation may be used to determine the mean and variance of the total prediction error and also whether it can be well-represented by a Gaussian probability distribution; for example, for the Virtual Seismologist ground-motion attenuation model [14], this approach has been used in [16,17] to show that the Gaussian distribution gives a good representation of the uncertainty in the total prediction error, despite the nonlinear form (given later in Eq. (27)), and that Eq. (8) gives a good approximation for its variance.

3. Decision model for activating mitigation measures

During the seismic event, a decision must be made between the two options: activate the mitigation measure or do nothing. This decision is based on the estimated value of the predictor and the uncertainty in the prediction process. This decision problem may be approached by the theory of sequential hypothesis testing [13], described as follows. A rule is given for making one of the following three decisions during the course of monitoring the seismic event where the hypothesis of interest is that the critical intensity measure will be exceeded:

- Accept the hypothesis
- Reject the hypothesis
- Continue and make an additional observation.

On the basis of the first observation (estimate) of the predictor, a decision is made. If the first option (accept) is chosen, then the mitigation measure is activated, while if the second option (reject) is chosen, then the monitoring process is ended. If the third option (continue) is chosen, then a second observation is made. Again, on the basis of the first two observations, a decision is made. If neither the first nor second option is chosen, then a third observation is made, and so on. This process is carried out until the first or second options are chosen, or the process is terminated because the shaking at the site has ended. A rule for making acceptance or rejection decisions consists in defining corresponding critical regions for the predictor. For each observation, the hypothesis is accepted or rejected if the estimate of the predictor lies in the corresponding critical region, otherwise the monitoring continues. In this work, the critical regions are defined in terms of thresholds for the probability of false and missed alarms, as described next.

3.1. Probabilities of making wrong decisions during the seismic event

The probabilities of false and missed alarms provide fundamental guidelines for the user's decision making during the seismic event, since they quantify the reliability of the information provided by the EEWS. While the event is occurring, the decision to take a mitigation action can be based on real-time monitoring of the probability of wrong decisions, focusing on the situation (false alarm or missed alarm) that the user is more concerned about. It will be demonstrated in the next subsection that this can also be done by monitoring whether the predicted intensity measure exceeds a time-varying warning threshold, $c(t)a$.

During a seismic event, the probability of false and missed alarms will be updated with time as more stations are triggered by the seismic waves and more data comes in from those that have already been triggered. This increase in data available will produce a decrease with time in the uncertainty of the predicted earthquake location and magnitude. Therefore, the prediction of the intensity measure can be updated and the characterization of its uncertainty, $\varepsilon_{\text{tot}}(t)$, will also vary with time. As a consequence, it is important to update the probability of false and missed alarms as the seismic event evolves.

The proposed procedure is as follows. The values of the probabilities $P_{\text{fa}}(t)$ and $P_{\text{ma}}(t)$ are evaluated as time goes by during an event and they are compared to the tolerable values, which may be established based on cost-benefit considerations (see the next subsection). The mitigation action is taken when either $P_{\text{fa}}(t)$ or $P_{\text{ma}}(t)$ reaches its tolerable value, β and α, respectively. This assumes that the time available is sufficient for activation of the mitigation measures. If the time available reaches the minimum time necessary for activation of the mitigation measure, no action will be taken if the probability of a wrong decision, evaluated at that time, is not acceptable, and the process will then be terminated for that event.

The procedure for calculating $P_{\text{fa}}(t)$ and $P_{\text{ma}}(t)$ is as follows. Recall that the predicted intensity measure $I\hat{M}$ is estimated from the attenuation model based on the predictions of earthquake magnitude and location provided by the EEWS and so it can be updated as a function of time during the event. On the other hand, the actual intensity measure value, IM, that

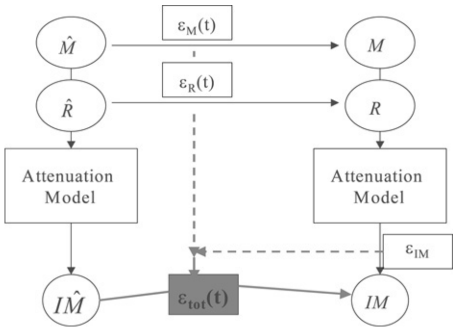

Fig. 3. Model of the EEWS prediction process during the seismic event.

will occur at the site is unknown. The predicted and actual values of the intensity measure differ by $\varepsilon_{tot}(t)$ as in Eq. (3):

$$IM = I\hat{M}(t) + \varepsilon_{tot}(t). \tag{10}$$

As shown in Fig. 3, the total error $\varepsilon_{tot}(t)$ is related to $\varepsilon_M(t)$, $\varepsilon_R(t)$ and ε_{IM}, which can be continually updated as additional information becomes available by using Bayesian updating, as in the Virtual Seismologist method [14]. Given the predicted value of $I\hat{M}$ at time t, the actual value IM is modeled by a Gaussian distribution with mean equal to the prediction $I\hat{M}(t)$ and with standard deviation equal to $\sigma_{tot}(t)$, which can be evaluated by uncertainty propagation of the error, as explained in Section 2.

The probability of a false alarm at time t is the probability of IM being less than the critical threshold, a, based on EEWS data, $D(t)$, received so far and given that the mitigation action is taken:

$$P_{fa}(t) = P[IM \le a \mid D(t), \text{action}]. \tag{11}$$

Since IM is modeled as a Gaussian distribution with mean equal to the predicted $I\hat{M}(t)$ (if there is a known bias in the prediction, it should be added to this mean) and with standard deviation $\sigma_{tot}(t)$, evaluated as a function of the updated uncertainties for the earthquake magnitude and location, it follows that:

$$P_{fa}(t) = \int_{-\infty}^{a} \frac{1}{\sigma_{tot}(t)\sqrt{2\pi}} \exp\left[\frac{-(IM - I\hat{M}(t))^2}{2\sigma_{tot}(t)^2}\right] \mathrm{d}IM$$
$$= \Phi\left(\frac{a - I\hat{M}(t)}{\sigma_{tot}(t)}\right) \tag{12}$$

where Φ is the standard Gaussian cumulative distribution function.

In a similar way, the probability of a missed alarm at time t is equal to the probability of IM being greater than the critical threshold based on EEWS data, $D(t)$, received so far and given that no mitigation action is taken:

$$P_{ma}(t) = P[IM > a \mid D(t), \text{no action}] \tag{13}$$

and

$$P_{ma}(t) = \int_{a}^{\infty} \frac{1}{\sigma_{tot}(t)\sqrt{2\pi}} \exp\left[\frac{-(IM - I\hat{M}(t))^2}{2\sigma_{tot}(t)^2}\right] \mathrm{d}IM$$
$$= 1 - \Phi\left(\frac{a - I\hat{M}(t)}{\sigma_{tot}(t)}\right). \tag{14}$$

Since the two conditions ($IM \le a$ and $IM > a$) are mutually exclusive and exhaustive, the probabilities $P_{fa}(t)$ and $P_{ma}(t)$ always sum to one. The analytical expressions in Eqs. (12) and (14) for calculating, respectively, the probabilities $P_{fa}(t)$ and $P_{ma}(t)$, allow one to calculate these values in real-time during an earthquake It remains to establish the tolerable levels of $P_{fa}(t)$ and $P_{ma}(t)$ that influence whether or not the mitigation action is to be taken.

3.2. Cost-benefit analysis for threshold design for action

Designing the threshold for action means defining the criteria for making a decision between taking a mitigation action or doing nothing. This threshold is defined by decision criteria based on the user's requirements, which could involve directly specifying the tolerable levels for the probabilities of wrong decisions or choosing them based on cost-benefit considerations. In the latter case, the decision criterion may be based on the minimization of the expected consequences over the two possible actions of performing the mitigation action or doing nothing.

A cost-benefit approach is analyzed here based on the details shown in Table 1 where C_{fa} is the cost of a false alarm and:

$$C_{ga} = C_{eq} - C_{save} \qquad C_{ma} = C_{eq} \qquad C_{gn} \simeq 0$$
$$P_{ga} = P[IM > a \mid D(t), \text{action}] = 1 - P_{fa} \tag{15}$$
$$P_{gn} = P[IM \le a \mid D(t), \text{no action}] = 1 - P_{ma}$$

where C_{eq} represents the expected costs due to the earthquake and C_{save} is the expected savings as a consequence of the activation of the mitigation measure. The three parameters C_{fa}, C_{eq} and C_{save} must be specified based on some prior analysis. If the mitigation action is taken, the expected cost is given by:

$$E\,[\text{cost} \mid \text{action}, D(t)] = C_{fa} \cdot P_{fa}(t) + C_{ga} \cdot P_{ga}(t)$$
$$= C_{fa} \cdot P_{fa}(t) + \left(C_{eq} - C_{save}\right) \cdot (1 - P_{fa}(t)). \tag{16}$$

On the other hand, if no action is taken, the expected cost is given by:

$$E\,[\text{cost} \mid \text{no-action}, D(t)] = C_{gn} \cdot P_{gn}(t) + C_{ma} \cdot P_{ma}(t)$$
$$= C_{eq} \cdot P_{ma}(t). \tag{17}$$

The decision criterion for deciding between the options, taking action or not, is represented by the minimum cost rule: Take the mitigation action if and only if:

$$E\,[\text{cost} \mid \text{no-action}, D(t)] > E\,[\text{cost} \mid \text{action}, D(t)] \tag{18}$$

that is,

$$C_{eq} \cdot P_{ma}(t) > C_{fa} \cdot P_{fa}(t) + (C_{eq} - C_{save}) \cdot (1 - P_{fa}(t))$$
$$= (C_{save} + C_{fa} - C_{eq}) \cdot P_{fa}(t) + (C_{eq} - C_{save}). \tag{19}$$

Since $P_{fa}(t) + P_{ma}(t) = 1$, Eq. (19) implies that the probability of a false alarm is tolerable if and only if:

$$P_{fa}(t) < \beta = \frac{C_{save}}{C_{fa} + C_{save}}. \tag{20}$$

Table 1
Cost-benefit analysis for threshold design

Mitigation action taken?	Cost for case $IM \leq a$	Cost for case $IM > a$
Yes	False alarm (false positive) cost: C_{fa}	Good alarm (true positive) cost: C_{ga}
No	Good no alarm (true negative) cost: C_{gn}	Missed alarm (false negative) cost: C_{ma}

Similarly, no action is to be taken if and only if:

$$C_{eq} \cdot P_{ma}(t) < (C_{save} + C_{fa} - C_{eq}) \cdot P_{fa}(t) + (C_{eq} - C_{save}) \tag{21}$$

so it follows that the probability of a missed alarm is tolerable if and only if:

$$P_{ma}(t) < \alpha = \frac{C_{fa}}{C_{fa} + C_{save}}. \tag{22}$$

As expected, $\alpha + \beta = 1$, which directly exhibits the trade-off between the threshold probabilities that are tolerable for false and missed alarms. If the threshold β is reduced to make false alarms less likely, then the threshold α for missed alarms becomes correspondingly larger.

The thresholds for the probabilities of false and missed alarms becoming intolerable can also be expressed in terms of a threshold for the predicted intensity measure $\hat{IM}(t)$. For the case of a missed alarm, the condition for taking the mitigation action is that $P_{ma}(t) > \alpha$ and so Eq. (14) gives the time-varying expression for the threshold as follows:

$$P_{ma}(t) > \alpha \Leftrightarrow \hat{IM}(t) > a\left[1 - \frac{\sigma_{tot}(t)\,\Phi^{-1}(1-\alpha)}{a}\right] = c_{ma}(t) \cdot a. \tag{23}$$

Therefore, the criterion for taking action based on the probability of a missed alarm becoming unacceptable is $\hat{IM}(t) > c_{ma}(t) \cdot a$ where:

$$c_{ma}(t) = 1 - \frac{\sigma_{tot}(t) \cdot \Phi^{-1}(1-\alpha)}{a}. \tag{24}$$

The mitigation action is also taken if the probability of a false alarm becomes acceptable and so based on Eq. (12):

$$P_{fa}(t) < \beta \Leftrightarrow \hat{IM}(t) > a\left[1 - \frac{\sigma_{tot}(t)\,\Phi^{-1}(\beta)}{a}\right] = c_{fa}(t) \cdot a \tag{25}$$

that is, the mitigation action is taken if $\hat{IM}(t) > c_{fa}(t) \cdot a$ where:

$$c_{fa}(t) = 1 - \frac{\sigma_{tot}(t) \cdot \Phi^{-1}(\beta)}{a}. \tag{26}$$

Therefore, taking action based on the exceedance of the predictor above these time-varying thresholds is equivalent to monitoring the probabilities of false and missed alarms and taking action when $P_{fa}(t)$ falls below the tolerable level β or $P_{ma}(t)$ exceeds the tolerable level α. Since the tolerable alarm probabilities sum up to one, the alarm probabilities will reach their critical thresholds at the same time, so one can choose to monitor either $P_{fa}(t)$ or $P_{ma}(t)$. Similarly, since $\alpha + \beta = 1$, if the predictor $\hat{IM}(t)$ is monitored, the critical thresholds $c_{ma}(t)a$ and $c_{fa}(t)a$ are equal and so are reached at the same time.

4. Illustrative example

As an illustrative example, the theory is applied to the magnitude 6.5 San Simeon earthquake that occurred on the Central Coast of California on 22 December 2003 (other examples are given in [17,18]). Imagining that an EEWS was in place, we explore the performance of the theory presented here for decision making during an earthquake. For this application, we choose the Virtual Seismologist method [14] to give the earthquake predictive model (M_1 in Fig. 2) and the attenuation model (M_2 in Fig. 2) defined by:

$$IM = \log_{10} \text{PGA} = a_{VS}M - b_{VS}[R_1 + C(M)] - d_{VS}\log_{10}[R_1 + C(M)] + e_{VS} + \varepsilon_{VS} \tag{27}$$

where M is the magnitude; R_1 depends on the epicentral distance, R; $C(M)$ is a correction factor depending on magnitude; the residual term ε_{VS} is a zero-mean error term representing the prediction uncertainty and e_{VS} is a constant error which includes station corrections; the parameters $a_{VS}, b_{VS}, d_{VS}, e_{VS}$ were estimated in [14] during the calibration of the model from data for different soil types and ε_{VS} was taken as Gaussian with zero mean and standard deviation 0.243.

The first station to be triggered was the Parkfield station at 57.2 km from the epicenter at 9.5 s after the earthquake origin time. The first estimates of magnitude and location are provided by the Virtual Seismologist [14] based on the first 3 s of waveform recorded at Parkfield and these predictions are updated after 5.5, and 8 s when more data become available.

Using the temporal sequence of updated magnitude and location predictions given by [14] for the San Simeon earthquake, we estimate the peak ground acceleration in log 10 scale based on the attenuation equation (27) but ignoring ε_{VS}, which produces the predicted intensity measure $\hat{IM}(t)$ (the most probable value of $IM = \log_{10}$ PGA at time t). We consider the epicentral distance prediction as time-invariant since the error associated with the predicted shaking intensity is much less sensitive to the epicentral distance error than to the magnitude error [16]. As a consequence of this, the magnitude uncertainty is continually updated and its standard deviation is assumed to decrease as $1/\sqrt{N}$, where N is the number of stations contributing information, following [14]. Therefore, the total uncertainty is also continually updated as a function of the magnitude uncertainty and attenuation model uncertainty using Eq. (8), as additional information is provided by the EEWS. Then, based on this total uncertainty associated with the prediction, the probabilities of a false alarm and of a missed alarm are evaluated as a tool for decision making during the event (Eqs. (12) and (14)). The threshold for taking action is also estimated from Eqs. (24) and (26).

As an example, consider the case of a structure located at 50, 60 and 70 km (Structure 1, 2 and 3, respectively) from the

Table 2
San Simeon earthquake: Relevant times for target structures, where T_p and T_s are P-wave and S-wave arrival times, T_d is the time of first broadcast of information by EEWS and T_{exc} is the time of threshold exceedance

Relevant times	Target Structure 1 $R = 50$ km	Target Structure 2 $R = 60$ km	Target Structure 3 $R = 70$ km
T_p (s)	8	10	12.5
T_d (s)	12.5	12.5	12.5
T_{exc} (s)	14.5	15.5	17.5
T_s (s)	17	20	23

San Simeon epicenter, assuming that the critical threshold for *PGA* is $a = 0.025$ g (corresponding to $a = 1.4$ in $\log_{10}$ scale in cm\s\s) and a tolerable level of probability of false alarm of $\beta = 0.4$. The P-wave and S-wave arrival times at 50, 60 and 70 km, where the structures are located, are predicted by [14]. The P-waves arrive after 8 s from the earthquake origin time at Structure 1, located at an epicentral distance of 50 km, but the first estimates of magnitude and location are not available yet from the EEWS because the first seismic station is located in Parkfield at 57.2 km and has not been triggered by the P-waves yet. The first EEWS predictions are available 4.5 s after the P-wave arrival at Structure 1; therefore, there are 4.5 s available between the first EEWS information and the S-wave arrival time at Structure 1, which is expected 17 s after the earthquake origin time. This time is used to monitor in real-time the probability of a false alarm and the expected ground shaking intensity at the site and to compare these values with their thresholds. The thresholds are exceeded at around 14.5 s, which means that there is 2.5 s for activation of a security measure before the arrival of the S-waves (see Fig. 4(a)). The P-waves reach Structure 2 located at 60 km from the epicenter at around 10 s and after 2.5 s the first information is available from the EEWS. Based on this information, the probability of a false alarm and the expected ground shaking intensity is evaluated as a function of time. The threshold is exceeded at 15.5 s, providing 4.5 s for activation of a security measure before the S-waves arrive (see Fig. 4(b)). P-waves arrive at Structure 3 at 70 km at 12.5 s after the earthquake origin. Based on the probability of a false alarm and the expected ground shaking intensity calculated using EEWS information, the thresholds are exceeded at around 17.5 s, providing 5.5 s for initiating mitigation procedures before the S-waves arrive (see Fig. 4(c)). Relevant times are summarized in Table 2. Note that the probability of a false alarm and the predicted shaking intensity are distance dependent while the tolerable level β and the time-varying alarm threshold $c_{\text{fa}}(t)a$ are not, depending only on cost-benefit considerations. This behavior is evident in Fig. 4.

This illustrative example shows clearly the direct dependence of the warning time on the distance between the location of the "target" structure to protect and the epicenter, where an increase of the epicentral distance corresponds to an increase in the warning time available for security measure activation. In addition, the parameters that characterize the EEWS application, both the critical threshold, a, and the tolerable level of probability of false alarm, β, play a fundamental role in deciding when to activate a mitigation measure at a site. The measures that can be carried out depend on the warning time available; in particular, on the time available from the moment at which the decision to take action is made until the moment at which the seismic S-waves arrive at the location of interest. In the case of a few seconds warning time before the shaking, as in this illustrative example (Fig. 4), it is still possible to take such actions as to slow down trains [10], to switch traffic lights to red (as for the Lions Gate Bridge EEWS, Vancouver), to close valves in gas and oil pipelines, to release control rods in nuclear power plants [2], and to activate structural control systems. In addition, secondary hazards can be mitigated that are triggered by earthquakes but which take more time to develop, such as landslides, tsunamis and fires, by predicting the ground-motion parameters for the incoming seismic waves. This could be used, for example, to initiate the evacuation of endangered areas.

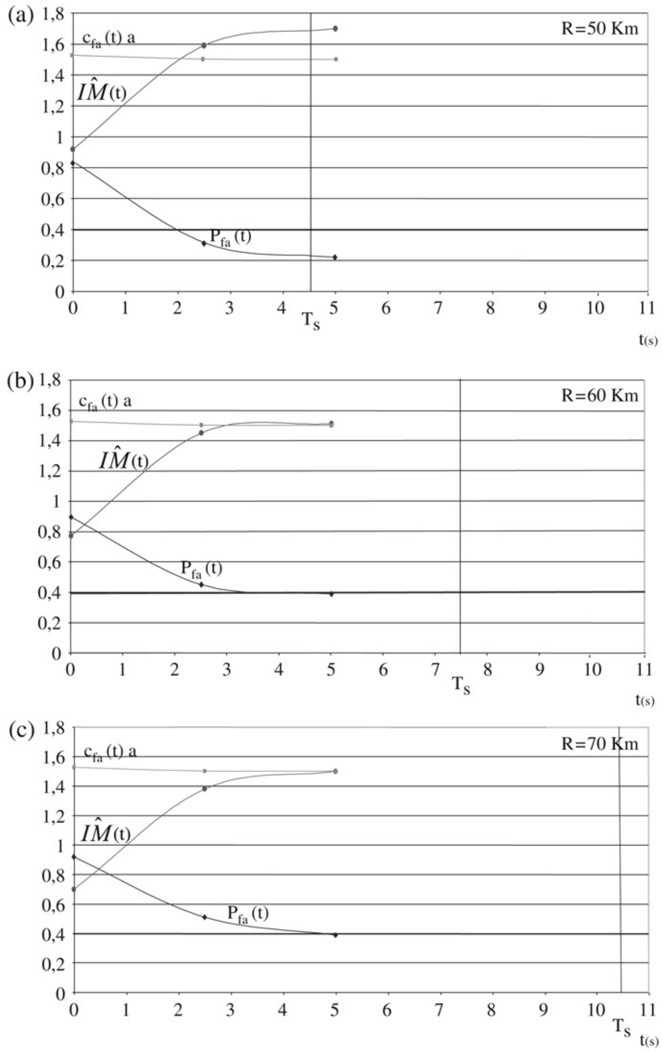

Fig. 4. $M = 6.5$ San Simeon, 2003: Decision making based on tolerable probability of false alarm of $\beta = 0.4$. The figure shows the probability of false alarm $P_{\text{fa}}(t)$, predicted intensity measure $\hat{IM}(t)$ and alarm threshold $c_{\text{fa}}(t)a$ as a function of time. The three plots correspond to cases of a target structure located at (a) 50 km, (b) 60 km and (c) 70 km from the epicenter. The time origin is the time of the first broadcast by EEWS which corresponds to 12.5 s (T_d of Table 2) after the earthquake origin time and T_s is the S-wave arrival time at the target structure.

5. Extension to other predictors: Real-time loss analysis

The decision model presented in the previous sections may be extended to other predictors that are of possible interest to the user, such as structural response parameters, damage or economic loss. For example, real-time loss analysis [19] allows the possibility of performing an automated loss analysis that provides expected loss estimates in place of the ground shaking intensity measure in the proposed decision theory. Before strong shaking initiates at a site of a facility, measures of expected losses, safety or operability of the facility might be calculated as decision variables based on the information about the seismic event that is provided by the EEWS. A near-real-time procedure for facility-specific loss analysis has been developed in [19] which builds on a performance-based earthquake engineering (PBEE) methodology. The goal in [19] is to give a probabilistic evaluation of damage, repair cost, safety and operability for an instrumented building (e.g. where accelerometers are located at the base), within a few minutes after the earthquake.

A similar methodology could be developed for estimating expected losses before shaking arrives at the site that is based on the PBEE methodology but using the real-time estimates of magnitude and epicentral distance from the EEWS to estimate the ground shaking intensity measure, *IM*, that is needed, as shown in Fig. 5 (adapted from [19]). In this figure, the data D is shown that comes from the stations of the seismic network and is processed in real-time by the central processing unit (or by the single nodes) by using the prediction models M_1 and M_2 that constitute the EEWS (see Fig. 2). The output of the EEWS is the probabilistic prediction of the ground shaking intensity *IM* expected at the site of the "target" facility, which can be determined as in Section 2. The error analysis presented there is necessary to evaluate the uncertainty associated with the prediction of *IM*. The intensity measure, *IM*, and its uncertainty are then used to establish $p(IM \mid D)$, a Gaussian distribution with mean equal to $\hat{IM} + \mu_{\text{tot}}(t)$ and standard deviation $\sigma_{\text{tot}}(t)$ (see Eqs. (9) and (8), respectively). The mean and standard deviation are updated with time, as more data become available. It is noted that for the chosen decision variable *DV*, $p(DV \mid IM)$ can be determined for a given facility prior to operation of the automated decision procedure (based on the Theorem of Total Probability). Then, in real-time, it can be integrated with $p(IM \mid D)$ from the EEWS to get $p(DV \mid D)$ (again based on the Theorem of Total Probability). This prior analysis to determine $p(DV \mid IM)$ can be based on an established PBEE procedure that requires a structural model along with fragility functions and repair cost distributions for each damageable assembly [20]. For the chosen decision variable *DV* (e.g. economic loss), the method will produce the probability distribution $p(DV \mid D)$ on which the decision whether to take action or not may be based; this is similar to the use of $p(IM \mid D)$ in the theory proposed earlier. The application of this proposed method to extend the automated decision procedure to other quantities of interest that may more closely represent the consequences of concern to the user, is left for future work.

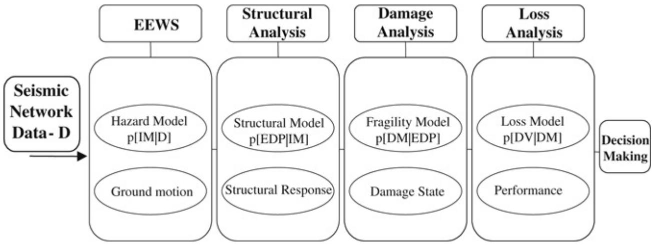

Fig. 5. Framework for PBEE-based early warning system (information flows from left to right).

6. Concluding remarks

Earthquake early warning systems represent an effective and promising tool for seismic risk mitigation by allowing mitigation measures to be taken at a facility when a warning is received. In the design of such systems, the trade-off between timeliness and reliability becomes a challenge to be addressed to enhance their effectiveness. Because of the short warning times available, an automated decision procedure is necessary for effective system implementation in which a predicted quantity of interest, such as the intensity of ground shaking, is continually monitored by updating it based on the incoming information from the early warning system.

The proposed decision model considers the probability of false alarms and missed alarms, and the expected consequences of taking a mitigation action or doing nothing, in the case of event detection. The decision of whether to take a mitigation action or not may be based on the probability of potentially making a wrong decision where the tolerable levels of probabilities of false and missed alarms may be chosen based on a cost-benefit analysis. The proposed procedure may be extended to the monitoring of other performance quantities of interest, such as the expected losses for the facility, as described in Section 5. The goal is to provide a decision-support tool that predicts the impact of the incoming seismic event so that a rational decision can be made whether or not to initiate mitigation measures in a timely manner.

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